

Mathematical Proofs

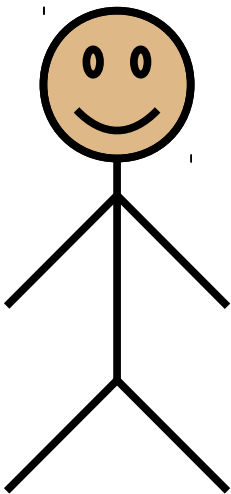
Outline for Today

- ***How to Write a Proof***
 - Synthesizing definitions, intuitions, and conventions.
- ***Proofs on Numbers***
 - Working with odd and even numbers.
- ***Universal and Existential Statements***
 - Two important classes of statements.
- ***Variable Ownership***
 - Who owns what?

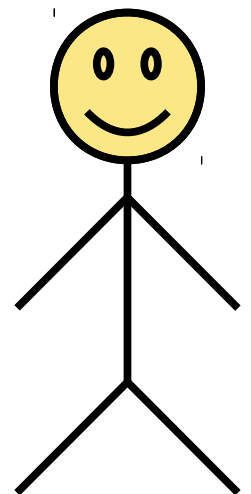
What is a Proof?

Proof as Dialog

- A mathematical proof is a dialog between two parties: a ***proof writer*** and a ***proof reader***.
 - The ***proof writer*** knows a mathematical fact.
 - The ***proof reader*** is honest but skeptical.
- The proof writer's job is to take the reader on a journey from ignorance to understanding.



Proof Writer (You)



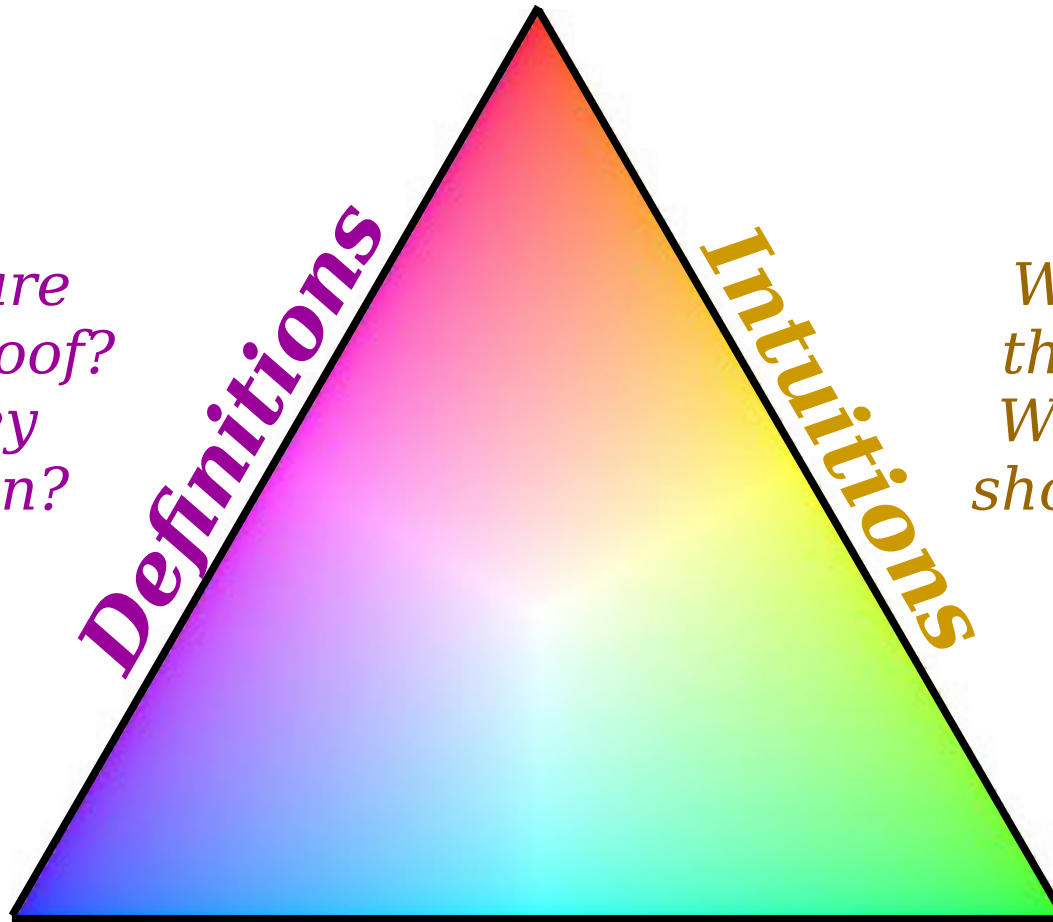
Proof Reader

*What terms are used in this proof?
What do they formally mean?*

Definitions

Intuitions

*What does this theorem mean?
Why, intuitively, should it be true?*

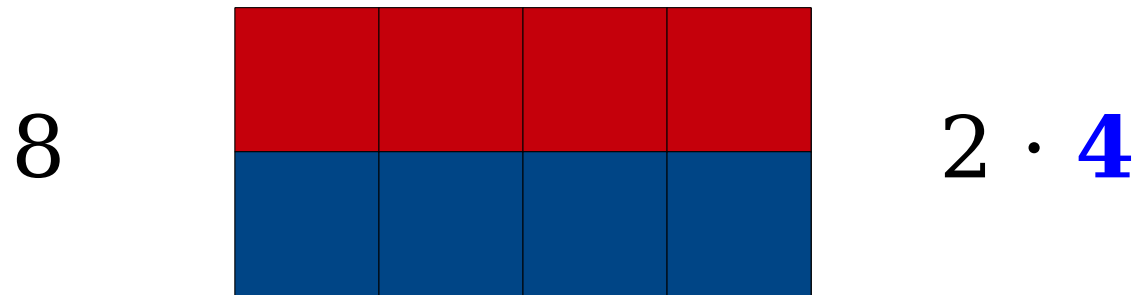
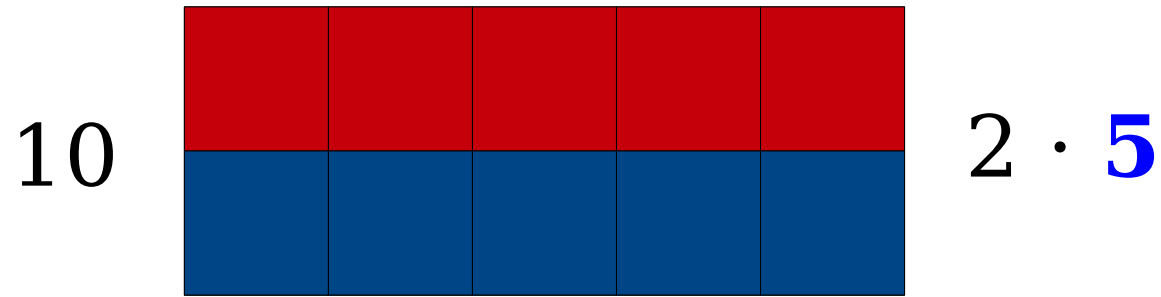


Conventions

*What is the standard format for writing a proof?
What are the techniques for doing so?*

Writing our First Proof

Theorem: If n is an even integer,
then n^2 is even.



An integer n is called ***even*** if there is an integer k where $n = 2k$.

Let's Try Some Examples!

$$2^2 = 4 = 2 \cdot \mathbf{2}$$

$$10^2 = 100 = 2 \cdot \mathbf{50}$$

$$0^2 = 0 = 2 \cdot \mathbf{0}$$

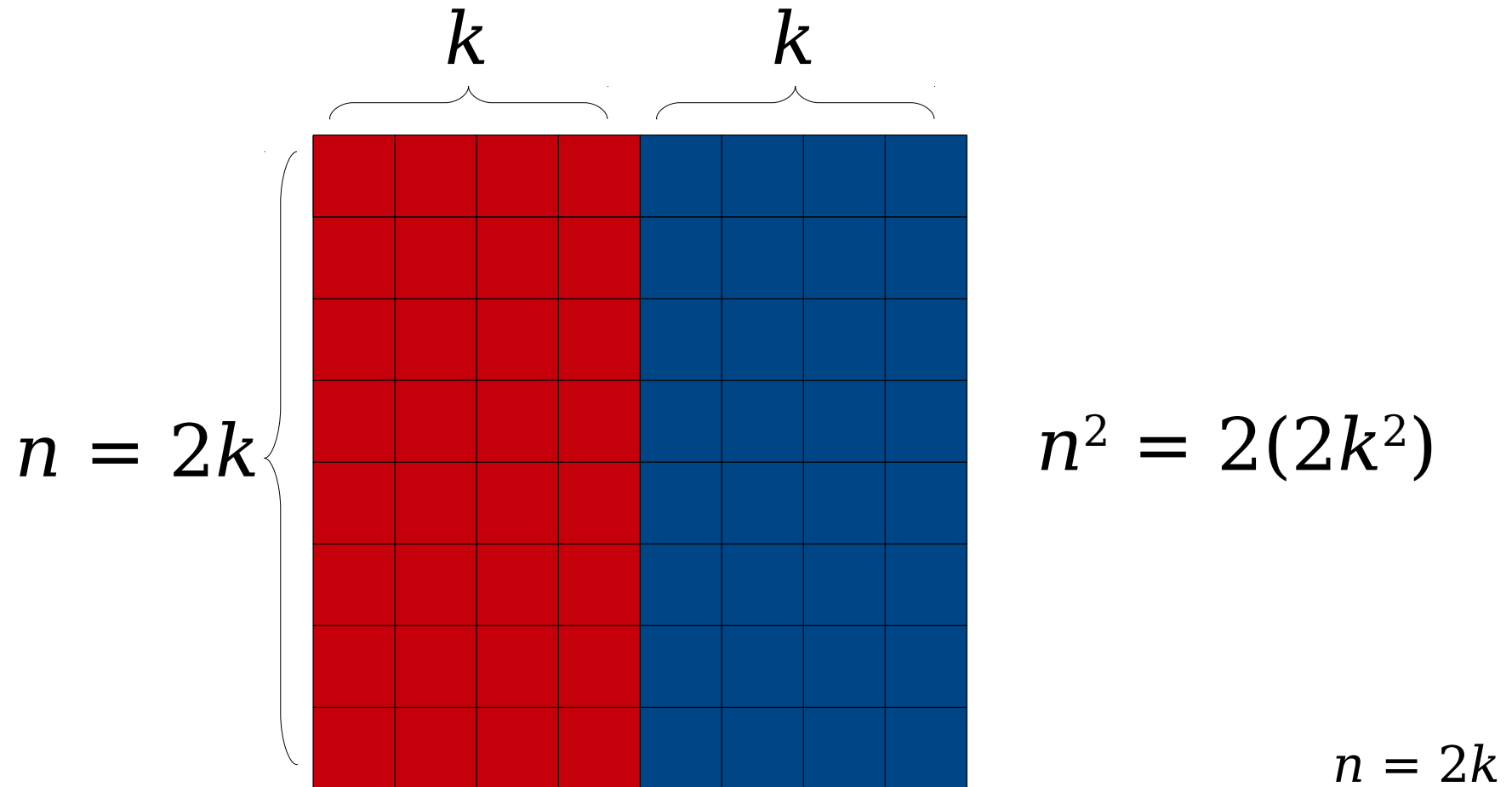
$$(-8)^2 = 64 = 2 \cdot \mathbf{32}$$

$$n^2 = 2 \cdot \mathbf{?}$$

What's the pattern? How do we predict this?

Theorem: If n is an even integer, then n^2 is even.

Let's Draw Some Pictures!



Theorem: If n is an even integer, then n^2 is even.

Our First Proof!

Theorem: If n is an even integer, then n^2 is even.

Proof: Assume n is an even integer. We want to show that n^2 is even.

Since n is even, there is some integer k such that $n = 2k$. This means that

$$\begin{aligned}n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2).\end{aligned}$$

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

Our First Proof!


Theorem: If n is an even integer, then n^2 is even.

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This symbol
means "end
of proof"



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Since n is even, there is some integer k such that $n = 2k$. This means that

To prove a statement of the form

“If P is true, then Q is true,”

start by asking the reader to assume that **P** is true.

From this, we see that $n^2 = (2k)^2 = 4k^2$ (namely, $2k^2$) which is even, which is

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To prove a statement of the form

“If P is true, then Q is true,”

we assume **P** is true, then need to show that **Q** is true. Here, we’re telling the reader where we’re headed.

From this,
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Our First Proof!

Theorem: If n is an even integer, then n^2 is even.

Proof: Assume n is an even integer. We want to show that n^2 is even.

Since n is even, there is some integer k such that $n = 2k$. This means that

This is the definition of an even integer. We need to use this definition to make this proof rigorous.

From this, (namely, $2k$) is even, which is what we wanted to show. ■

Our First Proof!

Theorem: If

Proof: Assume
show that

Since n is even,
that $n = 2k$

Notice how we use the value of k that we obtained above. Giving names to quantities, allows us to manipulate them. This is similar to variables in programs.

$$\begin{aligned}n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2).\end{aligned}$$

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

Our First Proof!

Theorem: If n is an even integer, then n^2 is even.

Proof: Assume n is an even integer. We want to show that n^2 is even.

Since
that n

Our ultimate goal is to prove that n^2 is even. This means that we need to find some m where

$n^2 = 2m$. Here, we're explicitly showing how we can do that.

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

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$$\begin{aligned}n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2)\end{aligned}$$

Hey, that's what we said we were going to do! We're done.

From this, we see that there is an integer m (namely, $2k^2$) where $n^2 = 2m$. Therefore, n^2 is even, which is what we wanted to show. ■

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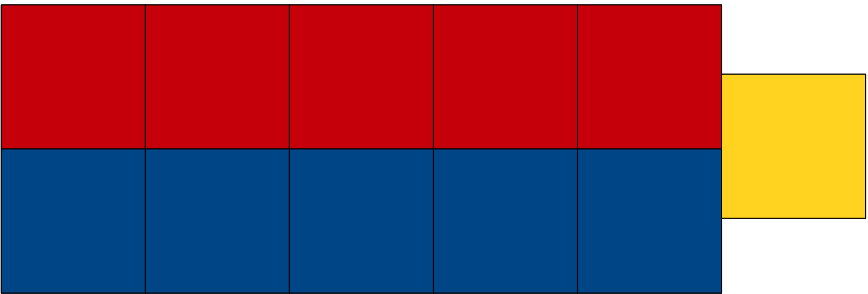
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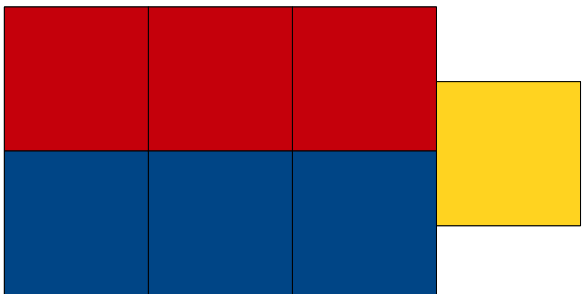
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Our Next Proof

Theorem: For any integers m and n , if m and n are odd, then $m + n$ is even.

11  $2 \cdot 5 + 1$

7  $2 \cdot 3 + 1$

1  $2 \cdot 0 + 1$

An integer n is called **odd** if there is an integer k where $n = 2k + 1$.

Going forward, we'll assume the following:

1. Every integer is either even or odd.
2. No integer is both even and odd.

Let's Try Some Examples!

$$1 + 1 = 2 = 2 \cdot \mathbf{1}$$

$$137 + 103 = 240 = 2 \cdot \mathbf{120}$$

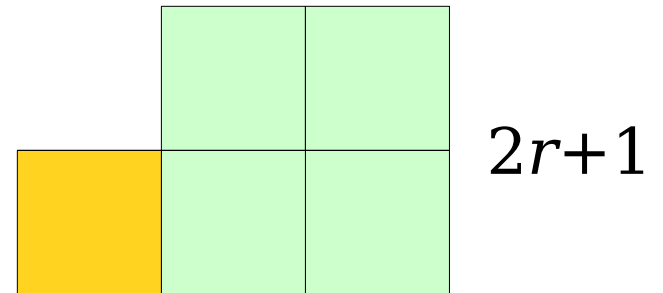
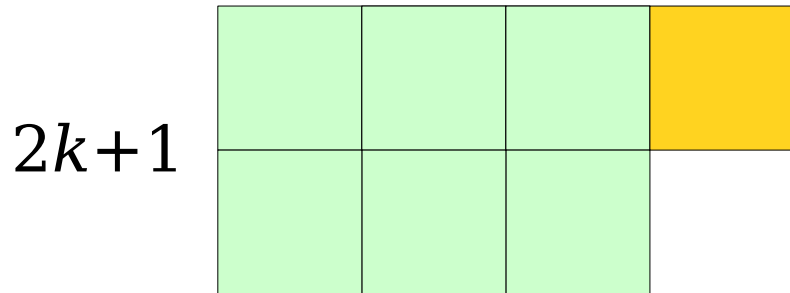
$$-5 + 5 = 0 = 2 \cdot \mathbf{0}$$

$$m + n = 2 \cdot \mathbf{?}$$

What's the pattern? How do we predict this?

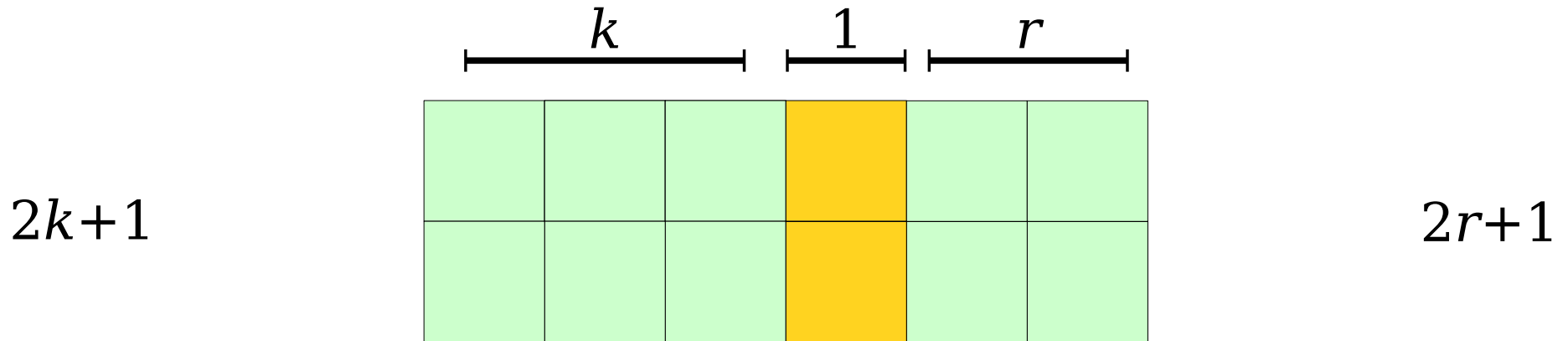
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Let's Do Some Math!



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Let's Do Some Math!



$$(2k+1) + (2r+1) = 2(k + r + 1)$$

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Theorem: For any integers m and n , if m and n are odd, then $m + n$ is even.

Proof: Consider any arbitrary integers m and n where m and n are odd. We need to show that $m + n$ is even.

Since m is odd, we know that there is an integer k where

$$m = 2k + 1. \quad (1)$$

Similarly, because n is odd there must be some integer r such that

$$n = 2r + 1. \quad (2)$$

By adding equations (1) and (2) we learn that

$$\begin{aligned} m + n &= 2k + 1 + 2r + 1 \\ &= 2k + 2r + 2 \\ &= 2(k + r + 1). \end{aligned} \quad (3)$$

Equation (3) tells us that there is an integer s (namely, $k + r + 1$) such that $m + n = 2s$. Therefore, we see that $m + n$ is even, as required. ■

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By adding equation

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We ask the reader to make an *arbitrary choice*. Rather than specifying what m and n are, we're signaling to the reader that they could, in principle, supply any choices of m and n that they'd like.

By letting the reader pick m and n arbitrarily, anything we prove about m and n will generalize to all possible choices for those values.

Theorem: For any integers m and n , if m and n are odd, then $m + n$ is even.

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To prove a statement of the form

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$$= 2k + 2r + 2$$

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after assuming P is true, you need to show that Q is true.

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Theorem: For any integers m and n , if m and n are odd, then $m + n$ is even.

Proof: Consider any odd. We need to show that $m + n$ is even. Since m is odd, we

Numbering these equalities lets us refer back to them later on, making the flow of the proof a bit easier to understand.

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Similarly, because n is odd there must be some integer r such that

$$(2)$$

This is a complete sentence! Proofs are expected to be written in complete sentences, so you'll often use punctuation at the end of formulas.

We recommend using the "mugga mugga" test - if you read a proof and replace all the mathematical notation with "mugga mugga," what comes back should be a valid

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$$+ 2r + 1$$

$$+ 2$$

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Some Little Exercises

- Here's a list of other theorems that are true about odd and even numbers:
 - **Theorem:** The sum and difference of any two even numbers is even.
 - **Theorem:** The sum and difference of an odd number and an even number is odd.
 - **Theorem:** The product of any integer and an even number is even.
 - **Theorem:** The product of any two odd numbers is odd.
- Going forward, we'll just take these results for granted. Feel free to use them in the problem sets.
- If you'd like to practice the techniques from today, try your hand at proving these results!

Universal and Existential Statements

Theorem: For any odd integer n ,
there exist integers r and s where $r^2 - s^2 = n$.

Theorem: For any odd integer n ,
there exist integers r and s where $r^2 - s^2 = n$.

This result is true for every
possible choice of odd integer n .
It'll work for $n = 1$, $n = 137$, $n =$
 103 , etc.

Theorem: For any odd integer n ,
there exist integers r and s where $r^2 - s^2 = n$.

We aren't saying this is true for every choice of r and s . Rather, we're saying that *somewhere out there* are choices of r and s where this works.

Universal vs. Existential Statements

- A ***universally-quantified statement*** is a statement of the form

For all x , [some-property] holds for x .

- We've seen how to prove these statements.
- An ***existentially-quantified statement*** is a statement of the form

There is some x where [some-property] holds for x .

- How do you prove an existentially-quantified statement?

Proving an Existential Statement

- Over the course of the quarter, we will see several different ways to prove an existentially-quantified statement of the form
There is an x where [some-property] holds for x .
- ***Simplest approach:*** Search far and wide, find an x that has the right property, then show why your choice is correct.

Let's Try Some Examples!

$$1 = \mathbf{1}^2 - \mathbf{0}^2$$

$$3 = \mathbf{2}^2 - \mathbf{1}^2$$

$$5 = \mathbf{3}^2 - \mathbf{2}^2$$

$$7 = \mathbf{4}^2 - \mathbf{3}^2$$

$$9 = \mathbf{5}^2 - \mathbf{4}^2$$

Theorem: For any odd integer n , there exist integers r and s where $r^2 - s^2 = n$.

Let's Try Some Examples!

$$1 = 2 \cdot \mathbf{0} + 1 = \mathbf{1}^2 - \mathbf{0}^2$$

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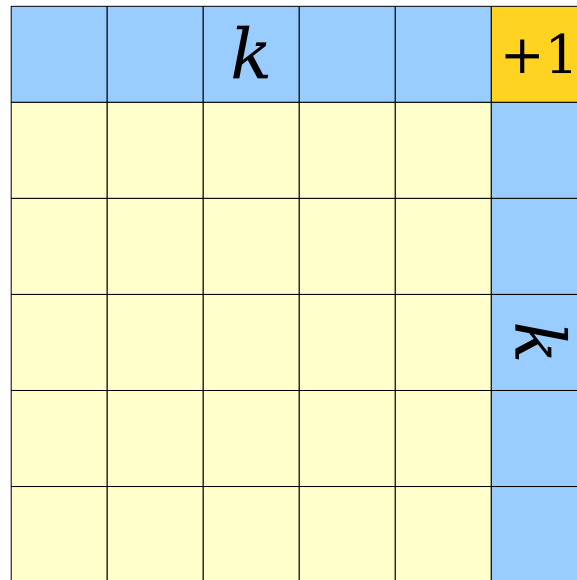
$$5 = 2 \cdot \mathbf{2} + 1 = \mathbf{3}^2 - \mathbf{2}^2$$

$$7 = 2 \cdot \mathbf{3} + 1 = \mathbf{4}^2 - \mathbf{3}^2$$

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Theorem: For any odd integer n ,
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Let's Draw Some Pictures!



$$(k+1)^2 - k^2 = 2k+1$$

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Proof: Let n be an arbitrary odd integer. We will show that there exist integers r and s where $r^2 - s^2 = n$.

Since n is odd, we know there is an integer k where $n = 2k + 1$. Now, let $r = k+1$ and $s = k$. Then we see that

$$\begin{aligned} r^2 - s^2 &= (k+1)^2 - k^2 \\ &= k^2 + 2k + 1 - k^2 \\ &= 2k + 1 \\ &= n. \end{aligned}$$

This means that $r^2 - s^2 = n$, which is what we needed to show. ■

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 $n = 2k + 1$
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We ask the reader to make an *arbitrary choice*. Rather than specifying what n is, we're signaling to the reader that they could, in principle, supply any choice n that they'd like.

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$$\begin{aligned} r^2 - s^2 &= (k+1)^2 - k^2 \\ &= k^2 + 2k + 1 - k^2 \\ &= 2k + 1 \\ &= n. \end{aligned}$$

As always, it's helpful to write out what we need to demonstrate with the rest of the proof.

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This means that $r^2 - s^2 = n$, which is what we wanted to show. ■

We're trying to prove an existential statement. The easiest way to do that is to just give concrete choices of the objects being sought out.

Theorem: For any odd integer n , there exist integers r and s where $r^2 - s^2 = n$.

Proof: Let n be an arbitrary odd integer. We will show that there exist integers r and s where $r^2 - s^2 = n$.

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This means that $r^2 - s^2 = n$, which is what we needed to show. ■

Time-Out for Announcements!

Working in Pairs

- Problem Set Zero is due this Friday at 4:00PM. It must be completed individually.
- After that, the remaining problem sets can be done individually or in pairs.
 - Each pair should make a single joint submission.
- We have advice about how to work effectively in pairs up on the course website – check the “Guide to Partners.”
- Want to work in a pair, but don’t know who to work with? You can:
 - Post a comment on the [***pinned partners thread***](#) on Ed.
 - Fill out [***this Google form***](#) and we’ll connect you with a partner on Monday.

Qt Creator Help Session

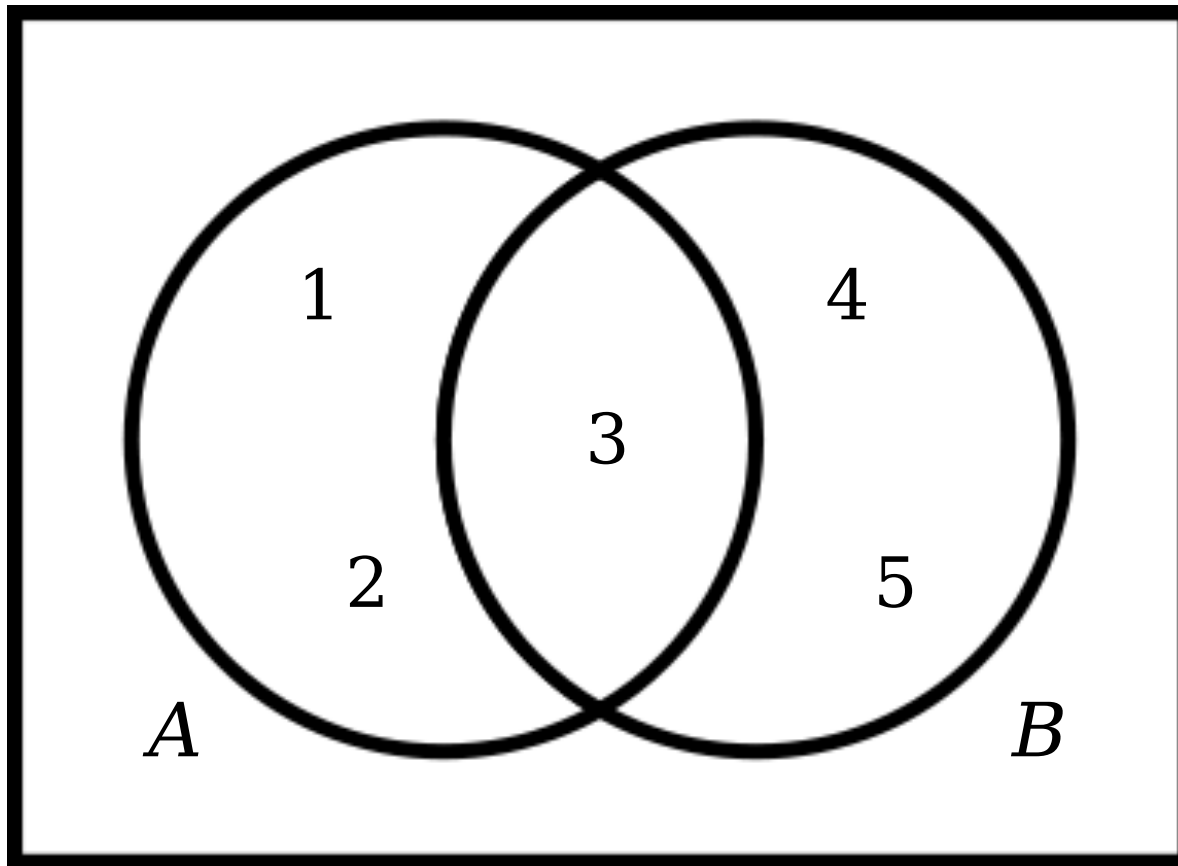
- The lovely CS106B staff are running a Qt Creator Help Session tomorrow evening if you're having trouble getting Qt Creator up and running on your system.
- Runs Thursday, January 11th from **5:00PM - 7:00PM** in Durand 353.
- SCPD students - please reach out to us if you need help setting things up. We'll do our best to help out.

Back to CS103!

A ***set*** is an unordered collection of distinct objects, which may be anything, including other sets.

Combining Sets

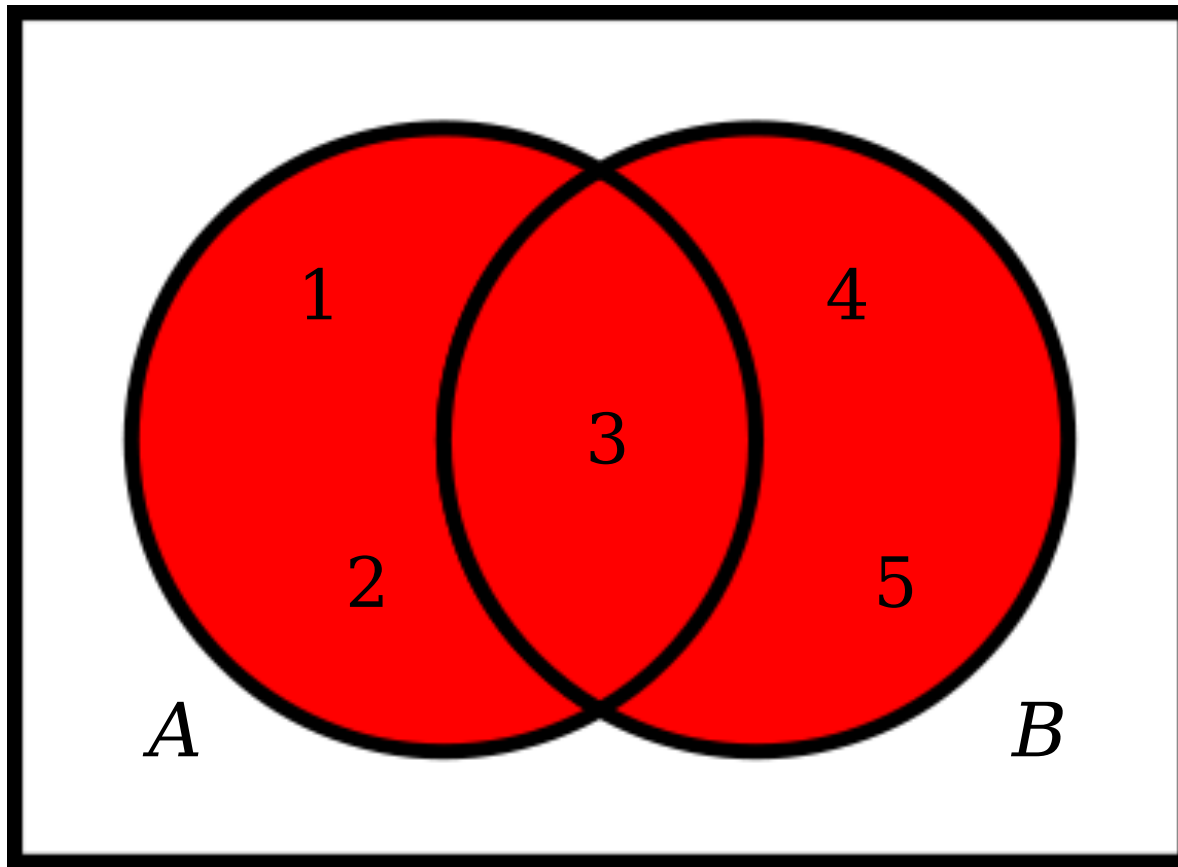
Venn Diagrams



$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

Venn Diagrams

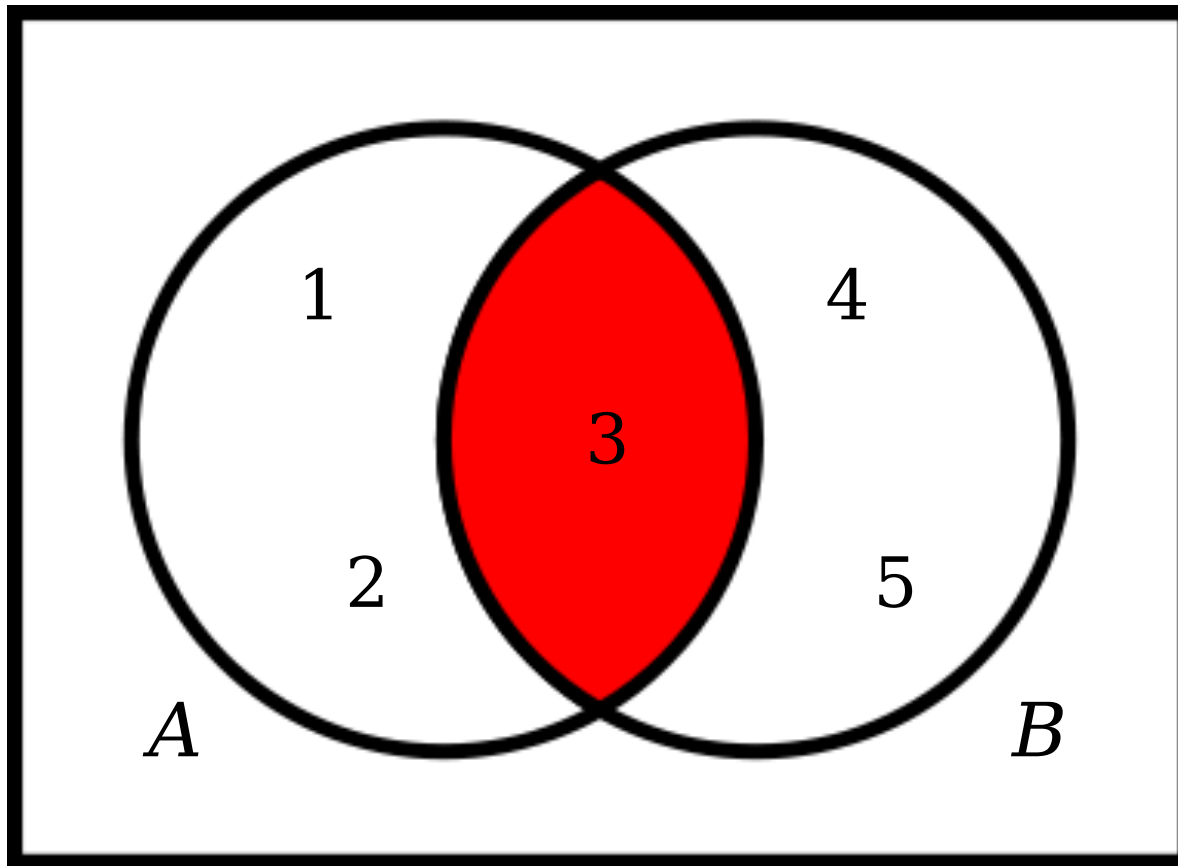


Union
 $A \cup B$
 $\{ 1, 2, 3, 4, 5 \}$

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

Venn Diagrams



Intersection

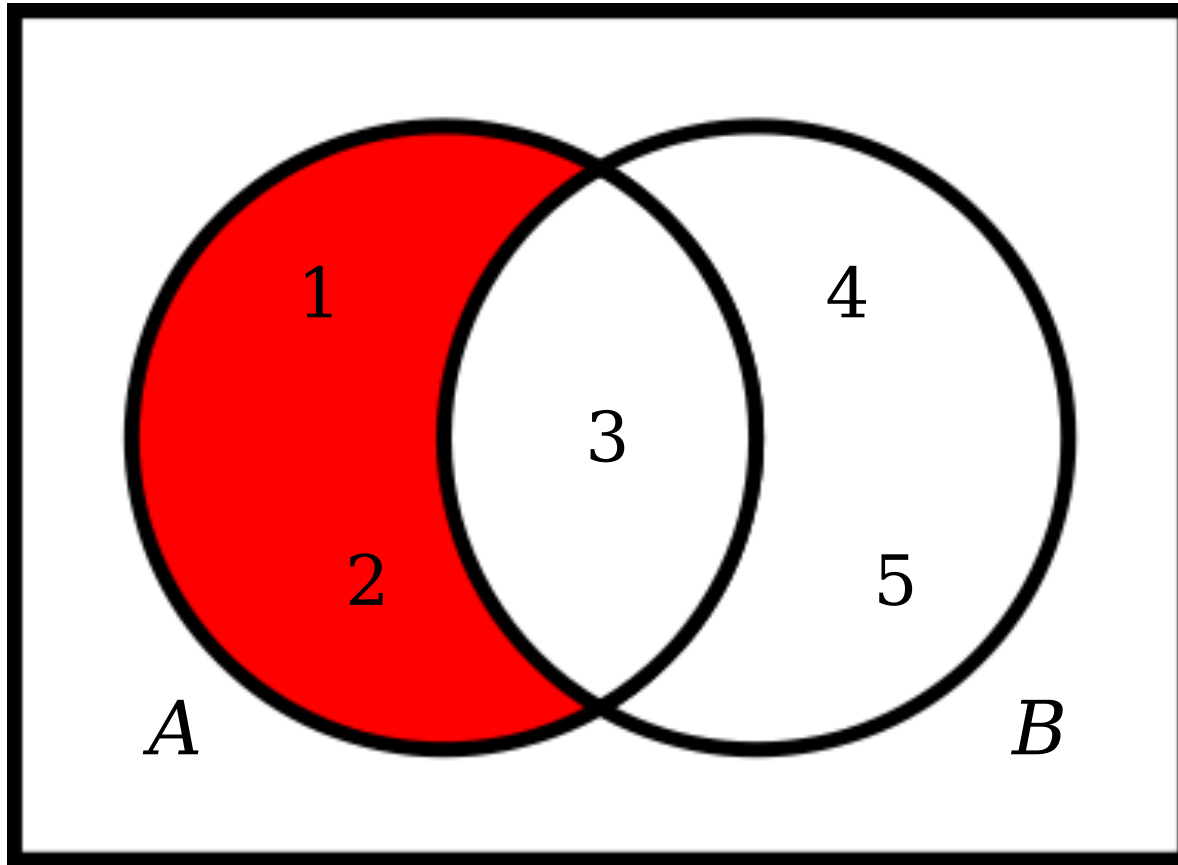
$$A \cap B$$

$$\{ 3 \}$$

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

Venn Diagrams



Difference

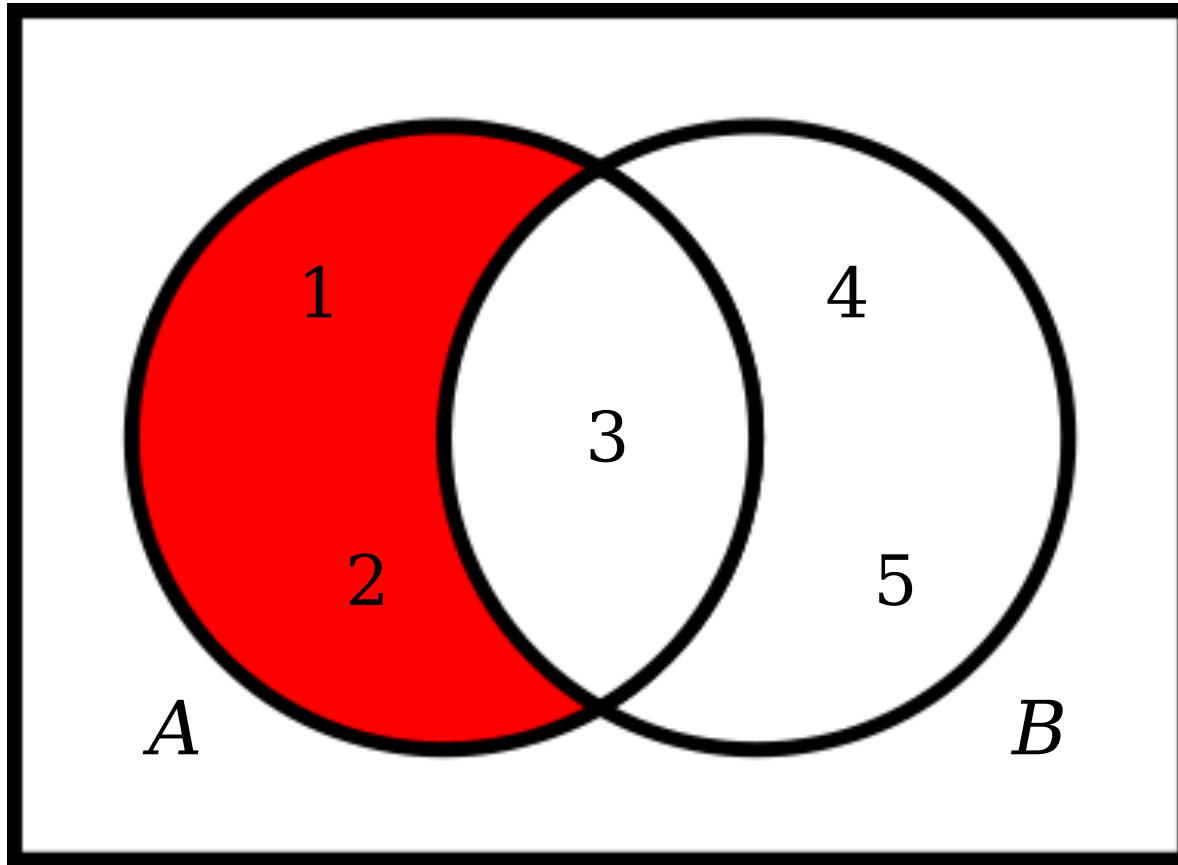
$$A - B$$

$$\{ 1, 2 \}$$

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

Venn Diagrams



Difference

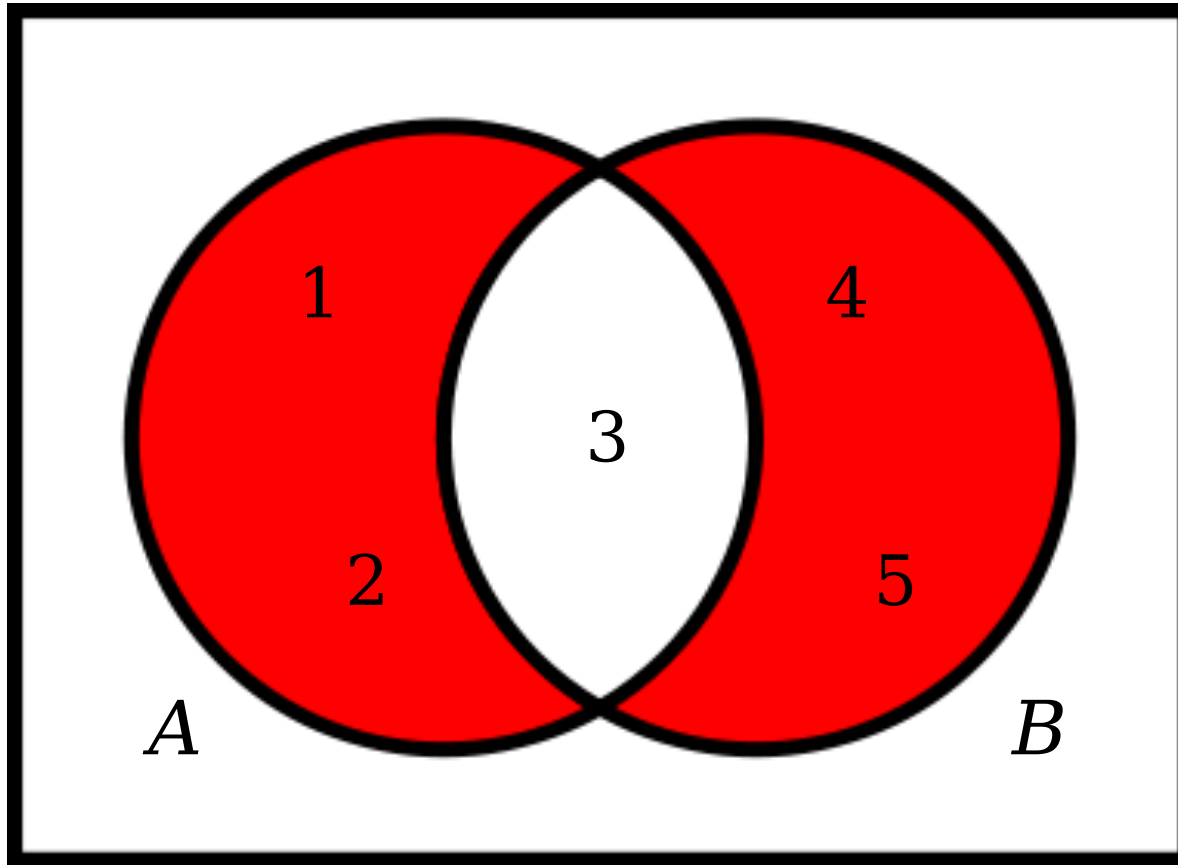
$$A \setminus B$$

$$\{ 1, 2 \}$$

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

Venn Diagrams

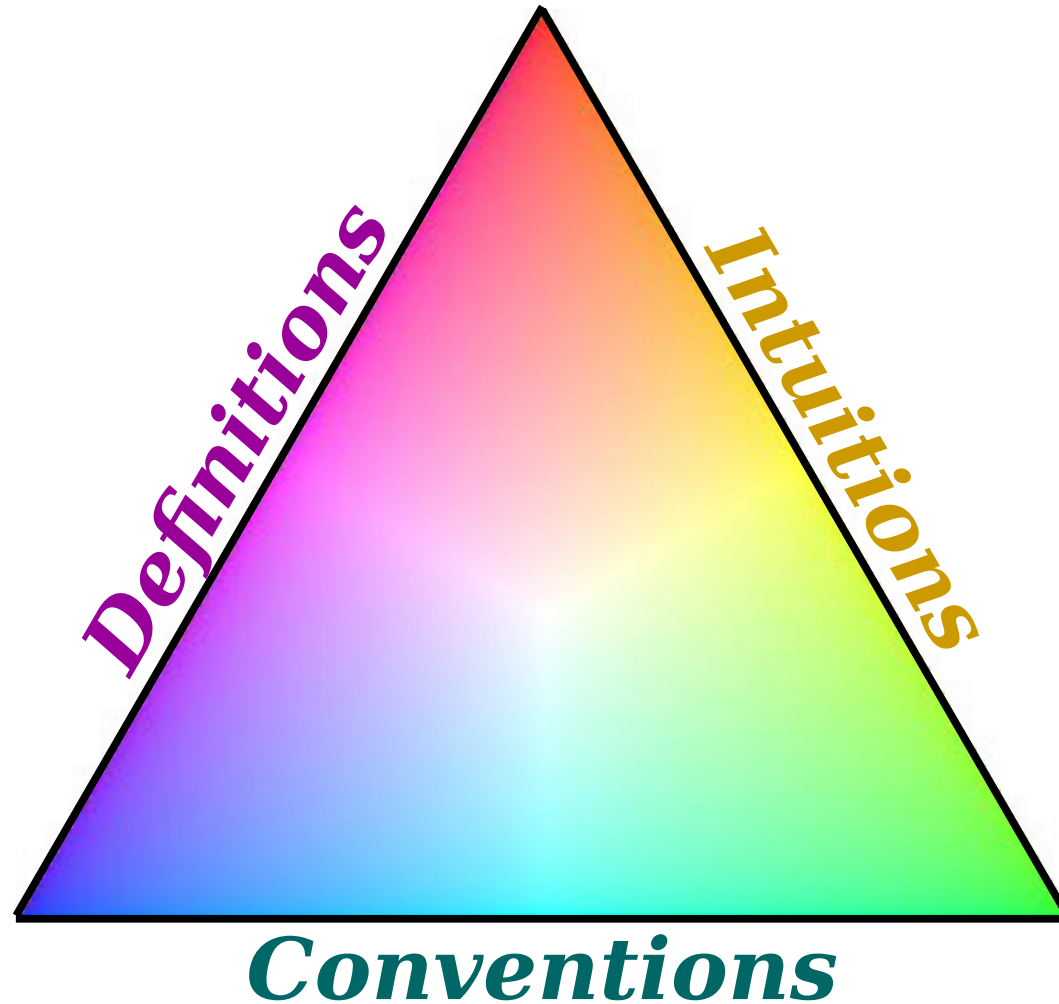


Symmetric
Difference
 $A \Delta B$
 $\{ 1, 2, 4, 5 \}$

$$A = \{ 1, 2, 3 \}$$

$$B = \{ 3, 4, 5 \}$$

To Recap



Writing a good proof requires a blend of definitions, intuitions, and conventions.

An integer n is **even** if there is an integer k where $n = 2k$.

An integer n is **odd** if there is an integer k where $n = 2k+1$.

Definitions tell us what we need to do in a proof.
Many proofs directly reference these definitions.

Let's Draw Some Pictures!

Let's Do Some Math!

Let's Try Some Examples!

Building intuition for results requires creativity,
trial, and error.

- Prove universal statements by making arbitrary choices.
- Prove existential statements by making concrete choices.
- Prove “If P , then Q ” by assuming P and proving Q .
- Write in complete sentences.
- Number sub-formulas when referring to them.
- Summarize what was shown in proofs by cases.
- Articulate your start and end points.

Mathematical proofs have established conventions that increase rigor and readability.

Your Action Items

- ***Read “Guide to \in and \subseteq ,” and “Guide to Partners.”***
 - These will be very useful for PS1.
- ***Finish and submit Problem Set 0.***
 - Don't put this off until the last minute!

Next Time

- ***Indirect Proofs***
 - How do you prove something without actually proving it?
- ***Mathematical Implications***
 - What exactly does “if P , then Q ” mean?
- ***Proof by Contrapositive***
 - A helpful technique for proving implications.
- ***Proof by Contradiction***
 - Proving something is true by showing it can't be false.

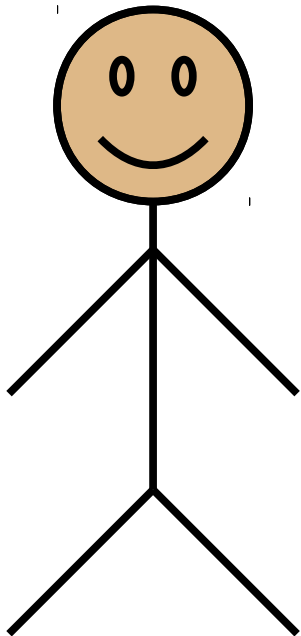
Appendix: *Proofs as Dialogs*

Proofs as a Dialog

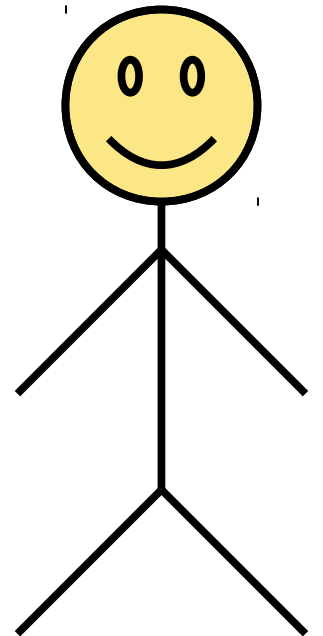
Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

Now, let $z = k - 34$.



Proof Writer (You)



Proof Reader

Proofs as a Dialog

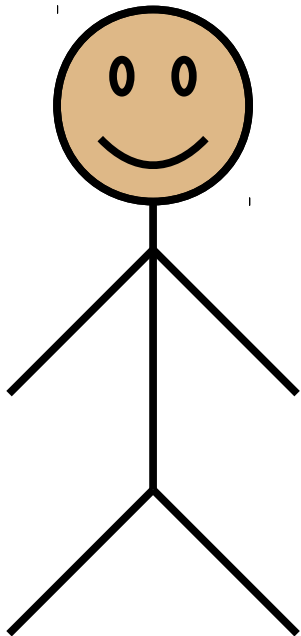
Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

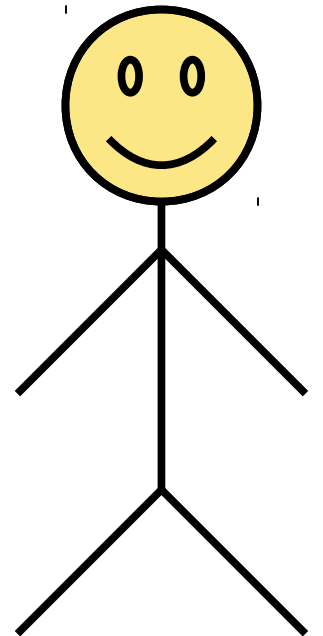
Now, let $z = k - 34$.

$$n = 137$$

Reader Picks



Proof Writer (You)



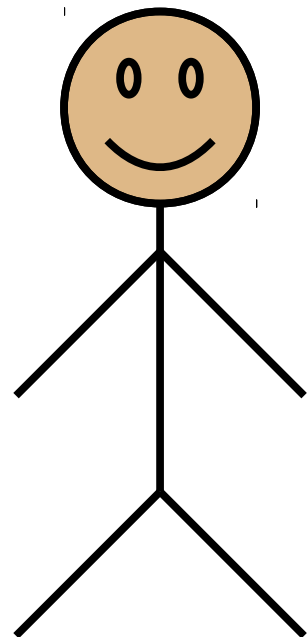
Proof Reader

Proofs as a Dialog

Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

Now, let $z = k - 34$.



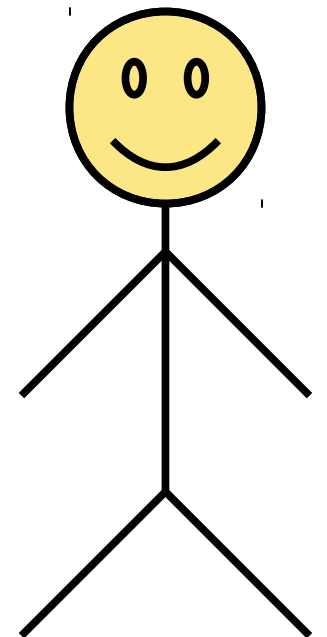
Proof Writer (You)

$k = 68$

Neither Picks

$n = 137$

Reader Picks



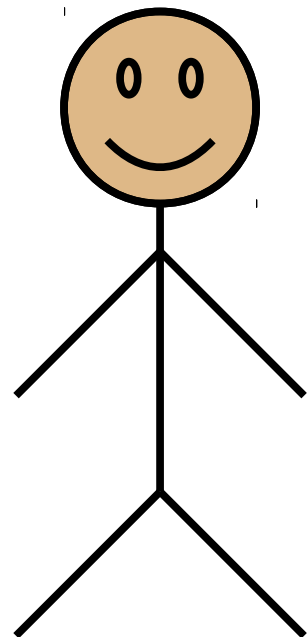
Proof Reader

Proofs as a Dialog

Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

Now, let $z = k - 34$.



Proof Writer (You)

$z = 34$

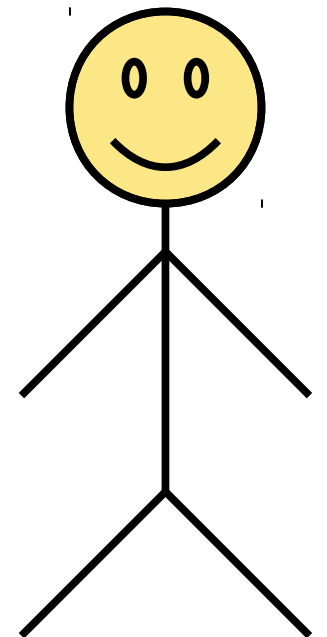
Writer Picks

$k = 68$

Neither Picks

$n = 137$

Reader Picks



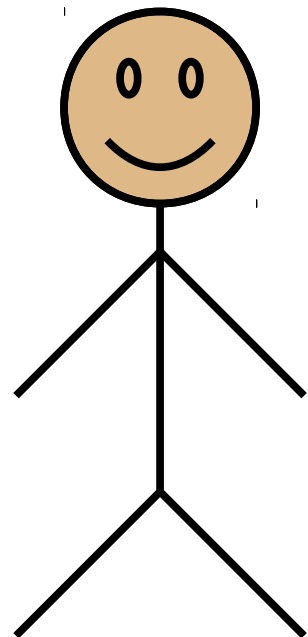
Proof Reader

Proofs as a Dialog

Let n be an arbitrary odd integer.

Since n is an odd integer, there is an integer k such that $n = 2k + 1$.

Now, let $z = k - 34$.



Proof Writer (You)

$z = 34$

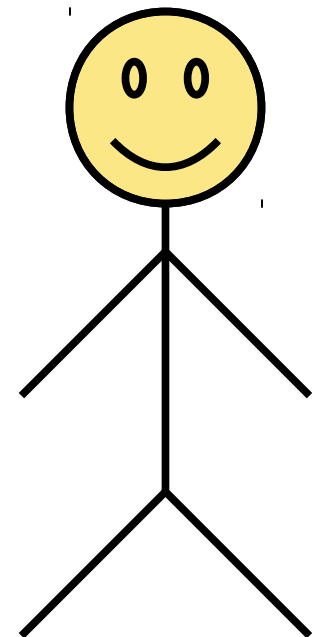
Writer Picks

$k = 68$

Neither Picks

$n = 137$

Reader Picks

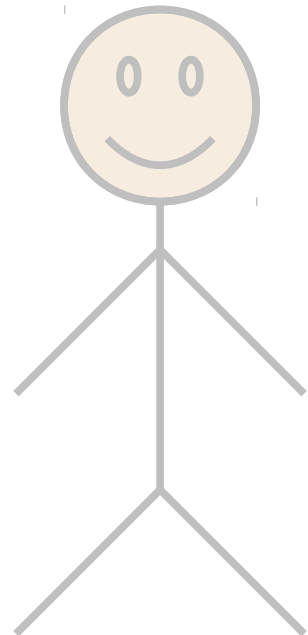


Proof Reader

Each of these variables has a distinct, assigned value.

Since Each variable was either picked by the reader, picked by the writer, or has a value that can be determined from other variables.

Now, let $z = k - 34$.



Proof Writer (You)

$z = 34$

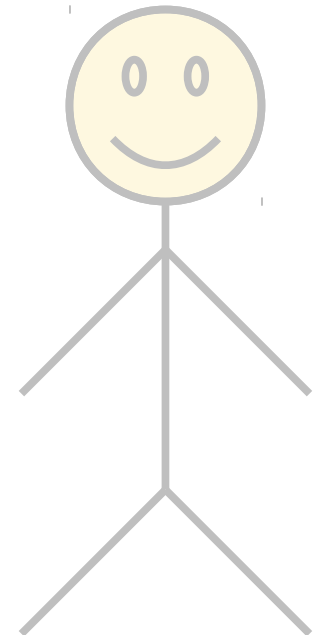
Writer Picks

$k = 68$

Neither Picks

$n = 137$

Reader Picks



Proof Reader

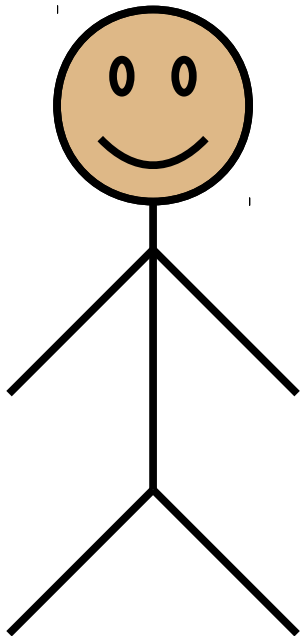
Who Owns What?

- The **reader** chooses and owns a value if you use wording like this:
 - Pick a natural number n .
 - Consider some $n \in \mathbb{N}$.
 - Fix a natural number n .
 - Let n be a natural number.
- The **writer** (you) chooses and owns a value if you use wording like this:
 - Let $r = n + 1$.
 - Pick $s = n$.
- **Neither** of you chooses a value if you use wording like this:
 - Since n is even, we know there is some $k \in \mathbb{Z}$ where $n = 2k$.
 - Because n is odd, there must be some integer k where $n = 2k + 1$.

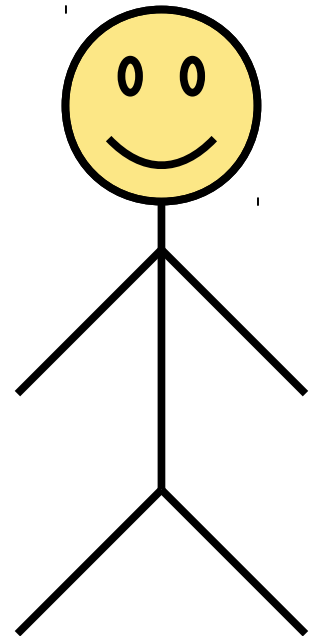
Proofs as a Dialog

Let x be an arbitrary even integer.

Then for any even x , we know that $x+1$ is odd.



Proof Writer (You)

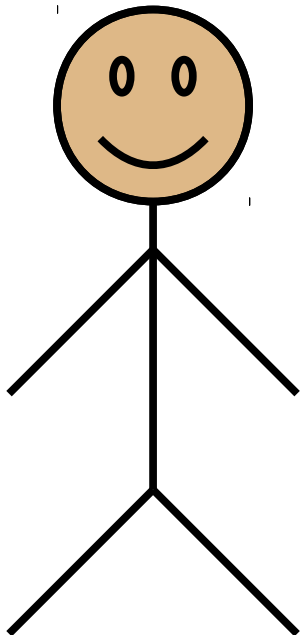


Proof Reader

Proofs as a Dialog

Let x be an arbitrary even integer.

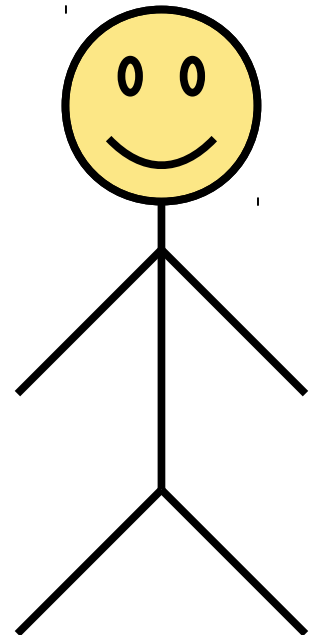
Then for any even x , we know that $x+1$ is odd.



Proof Writer (You)

$$x = 242$$

Reader Picks

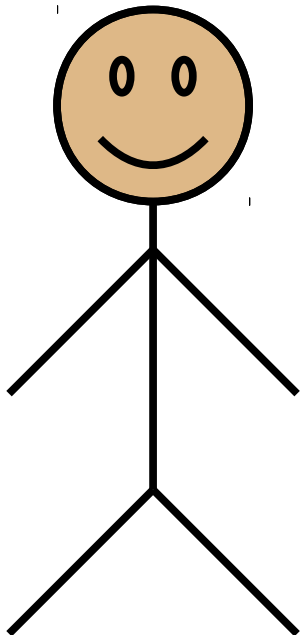


Proof Reader

Proofs as a Dialog

Let x be an arbitrary even integer.

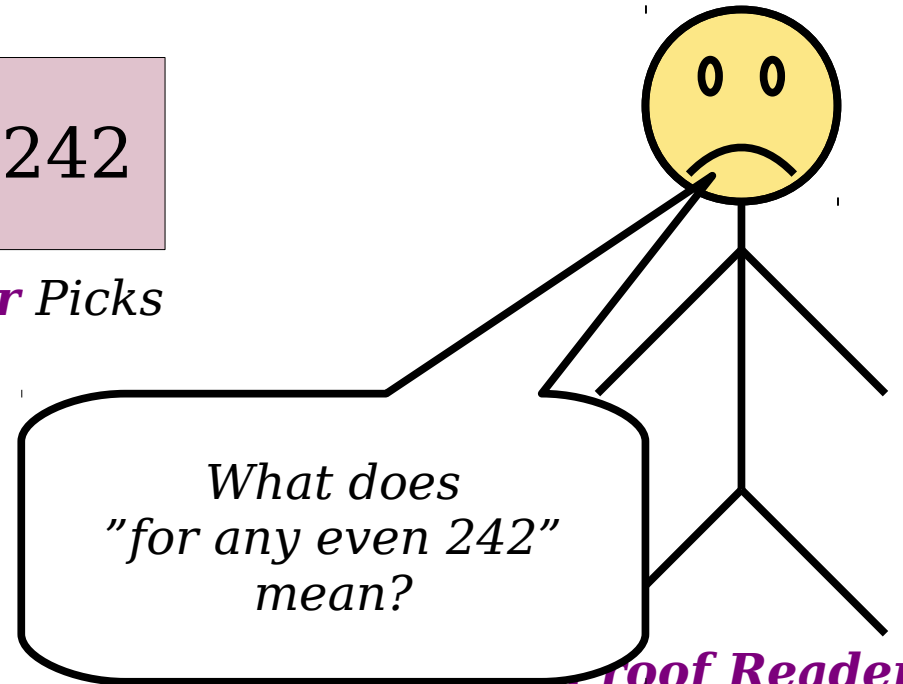
Then **for any even x** , we know that $x+1$ is odd.



Proof Writer (You)

$$x = 242$$

Reader Picks

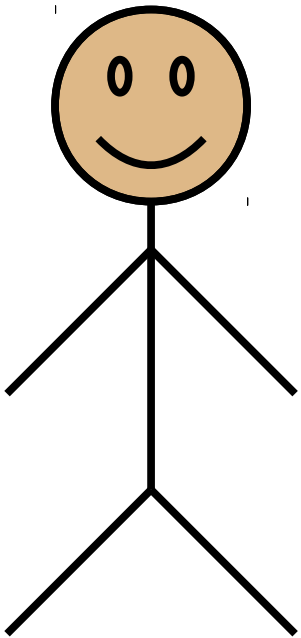


Proof Reader

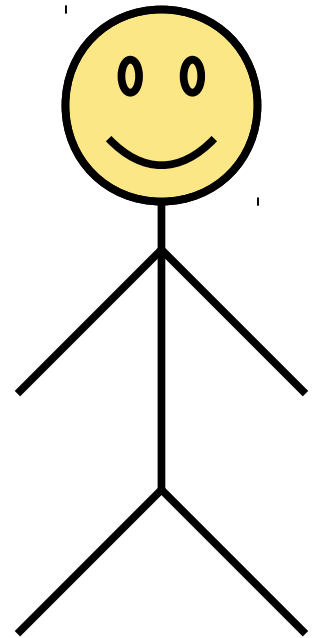
Proofs as a Dialog

Let x be an arbitrary even integer.

Since x is even, we know that $x+1$ is odd.



Proof Writer (You)

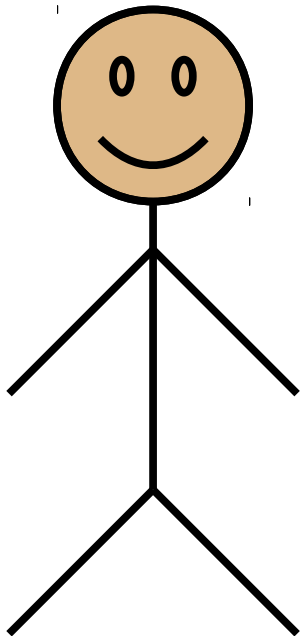


Proof Reader

Proofs as a Dialog

Let x be an arbitrary even integer.

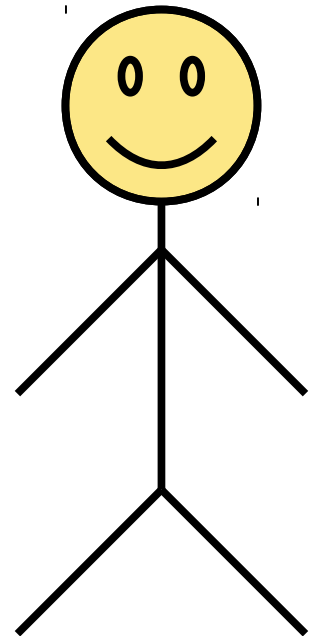
Since x is even, we know that $x+1$ is odd.



Proof Writer (You)

$$x = 242$$

Reader Picks

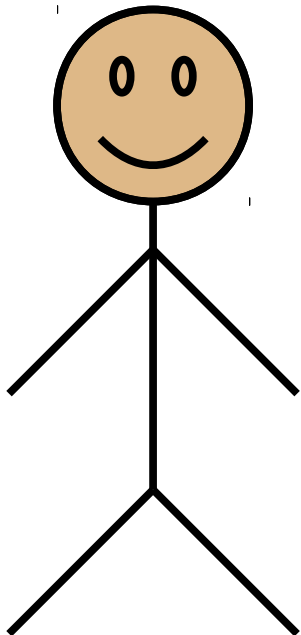


Proof Reader

Proofs as a Dialog

Let x be an arbitrary even integer.

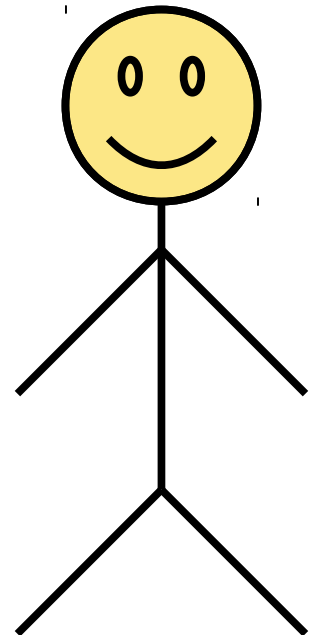
Since x is even, we know that $x+1$ is odd.



Proof Writer (You)

$$x = 242$$

Reader Picks



Proof Reader

Every variable needs a value.

***Avoid talking about “all x ” or “every x ”
when manipulating something
concrete.***

***To prove something is true for any
choice of a value for x , let the reader
pick x .***

Once you've said something like

Let x be an integer.
Consider an arbitrary $x \in \mathbb{Z}$.
Pick any x .

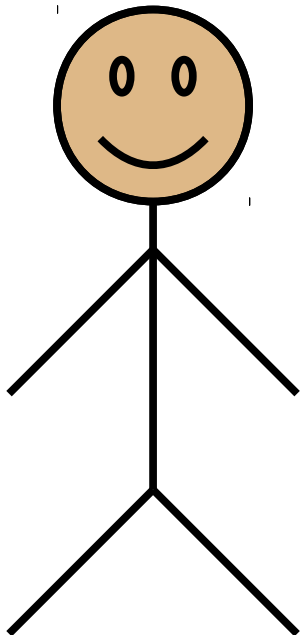
Do not say things like the following:

This means that ***for any*** $x \in \mathbb{Z} \dots$
So ***for all*** $x \in \mathbb{Z} \dots$

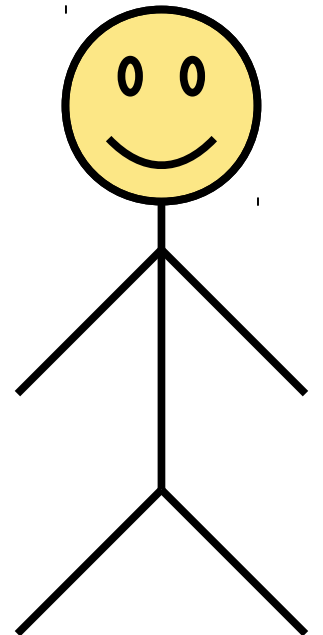
Proofs as a Dialog

! Pick two integers m and n where $m+n$ is odd. !

Let $n = 1$, which means that $m+1$ is odd.



Proof Writer (You)

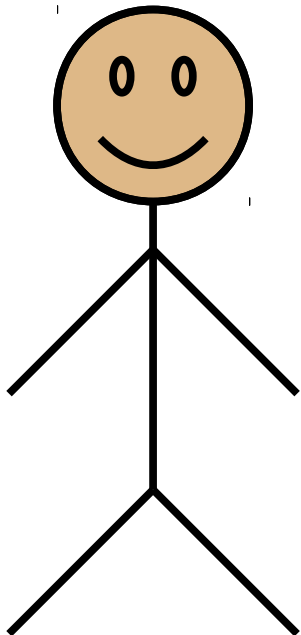


Proof Reader

Proofs as a Dialog

Pick two integers m and n where $m+n$ is odd.

Let $n = 1$, which means that $m+1$ is odd.



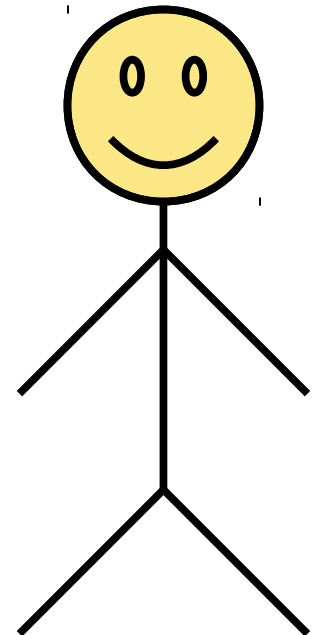
Proof Writer (You)

$$m = 103$$

Reader Picks

$$n = 166$$

Reader Picks

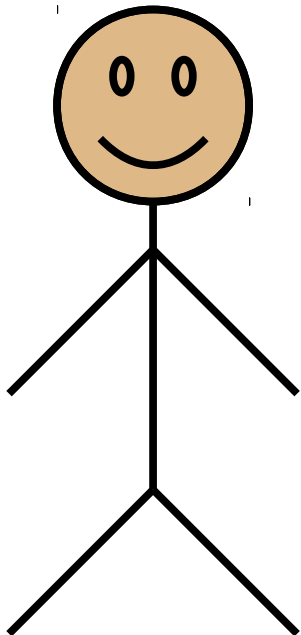


Proof Reader

Proofs as a Dialog

Pick two integers m and n where $m+n$ is odd.

Let $n = 1$, which means that $m+1$ is odd.



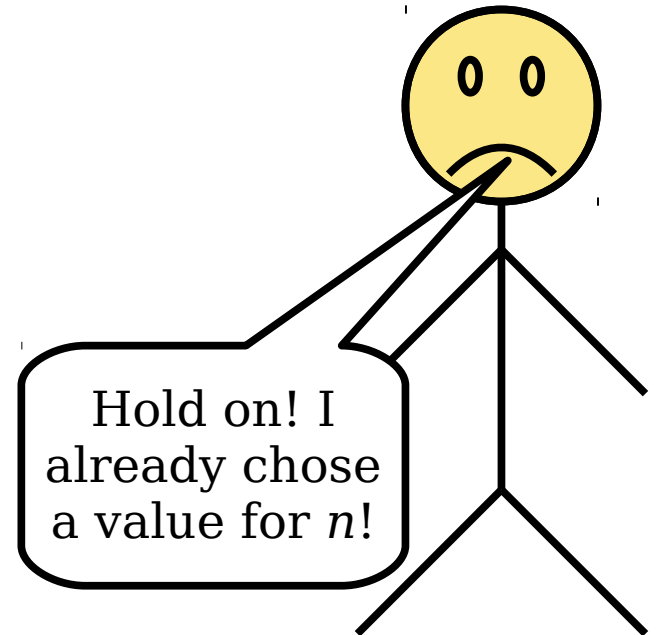
Proof Writer (You)

$$m = 103$$

Reader Picks

$$n = 166$$

Reader Picks

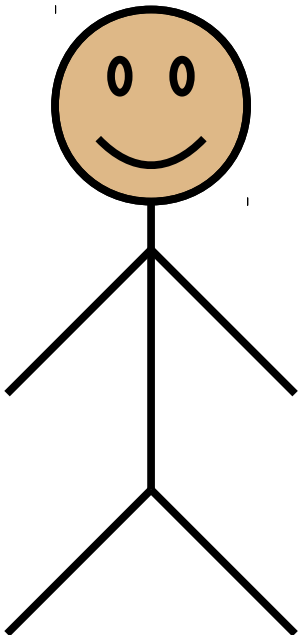


Proof Reader

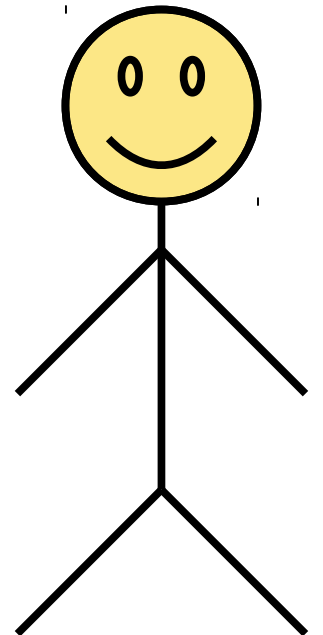
Proofs as a Dialog

Let $n = 1$.

Pick any integer m where $m+1$ is odd.



Proof Writer (You)

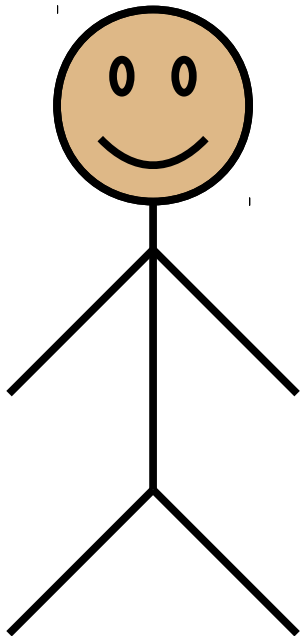


Proof Reader

Proofs as a Dialog

Let $n = 1$.

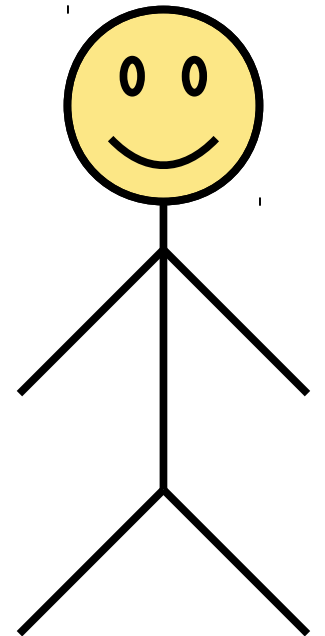
Pick any integer m where $m+1$ is odd.



Proof Writer (You)

$n = 1$

Writer Picks

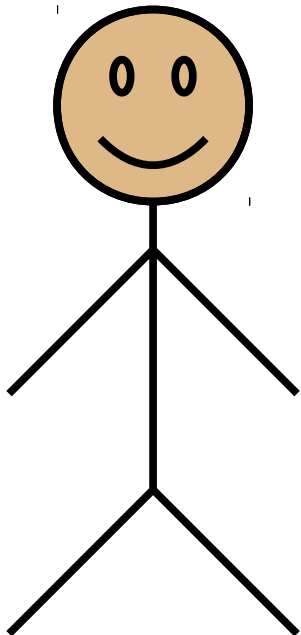


Proof Reader

Proofs as a Dialog

Let $n = 1$.

Pick any integer m where $m+1$ is odd.



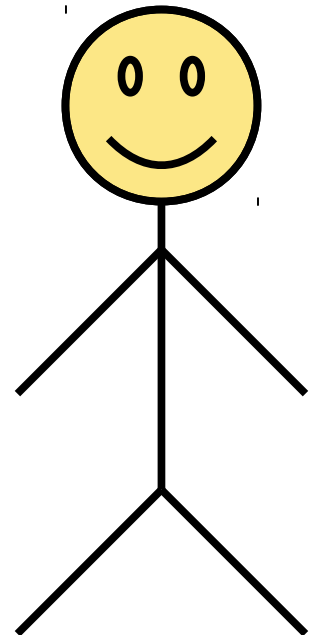
Proof Writer (You)

$m = 166$

Reader Picks

$n = 1$

Writer Picks



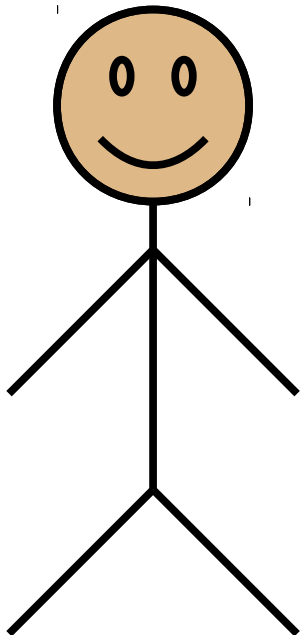
Proof Reader

Proofs as a Dialog

Do we even need n here?

Let $n = 1$.

Pick any integer m where $m+1$ is odd.



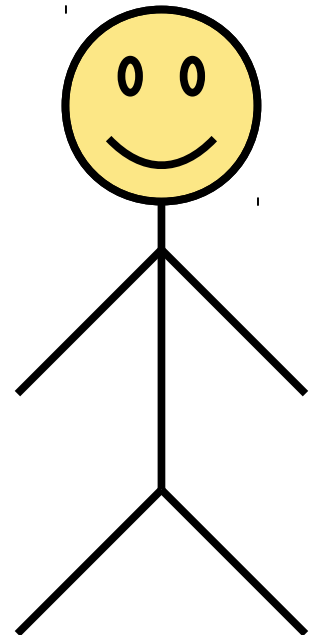
Proof Writer (You)

$m = 166$

Reader Picks

$n = 1$

Writer Picks



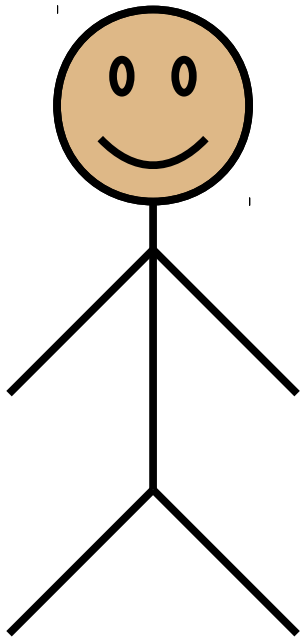
Proof Reader

Proofs as a Dialog

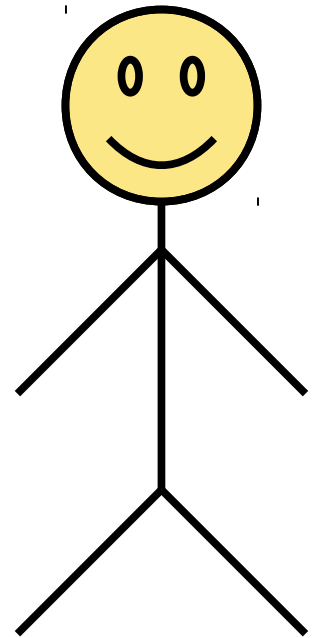
Pick any integer m where $m+1$ is odd.

$$m = 166$$

Reader Picks



Proof Writer (You)



Proof Reader

Be mindful of who owns what variable.

Don't change something you don't own.

***You don't always need to name things,
especially if they already have a name.***